UNIT-1(WAVE MECHANICS)

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MATTER WAVES -DE- BROGLIE WAVELENGTH

What is Quantum Mechanics?

Quantum mechanics is a branch of physics that explains the behavior of very tiny particles like electrons, atoms, and photons.

• Unlike classical physics, quantum mechanics deals with things that are too small to see.

• It shows that particles can act like both particles **and** waves.

• Quantum mechanics helps explain how atoms work, how light behaves, and how modern electronics like lasers and computers function.

• It is the foundation of modern physics and used in technologies like semiconductors and quantum computing.

2. MATTER WAVES

What are Matter Waves?

Matter waves are the wave-like nature of tiny particles such as electrons, protons, and even atoms.

• Proposed by **Louis de Broglie**, who said particles can behave like waves.

• This idea is called **wave-particle duality** – meaning every particle has a wavelength.

• The wavelength (λ) is given by $\lambda = h / p$, where

h = Planck's constant

p = momentum of the particle

• This wave nature was later confirmed by experiments like electron diffraction.

3 PROPERTIES OF MATTER WAVES

What are the Properties of Matter Waves?

Matter waves have unique properties that are different from classical particles:

• **Wavelength depends on momentum** – slower particles have longer wavelengths.

• **Not visible to the eye** – they are very tiny, only detectable through experiments.

• **Can interfere and diffract** – just like light waves, matter waves can create patterns.

• **Affected by observation** – observing a matter wave can change its behavior (related to the Heisenberg uncertainty principle).

• They show **quantum behavior**, helping us understand atomic and subatomic systems.

Here you go — explained in the same simple and clear format as before:

4. WAVE FUNCTION

What is a Wave Function?

A wave function is a mathematical function that describes the behavior of a particle in quantum mechanics.

• It is usually written as the symbol **ψ (psi)**.

• The wave function tells us **where a particle is likely to be** (its probability).

• It doesn't give exact answers like in classical physics, but **probabilities**.

• The square of the wave function, **$|\psi|^2$**, gives the **probability density** — the chance of finding the particle in a certain place.

• The wave function can change over time and space, depending on the situation.

PROPERTIES OF WAVE FUNCTION

What are the Properties of a Wave Function?

For a wave function to be valid in quantum mechanics, it must follow these rules:

• **Single-Valued** – It must give **only one value** at any point in space.

• **Continuous** – It should be smooth, **not jump suddenly** from one value to another.

• **Finite** – The value of the wave function must **not be infinite** anywhere.

• **Normalizable** – The total probability of finding the particle **must be 1**. This means:

$\int |\psi|^2 \, dx = 1$ (over all space)

• **Differentiable** – It should be possible to take the derivative of the wave function (needed for equations like Schrödinger's).

PHYSICAL SIGNIFICANCE

What is the Physical Significance of a Wave Function?

The wave function itself (ψ) doesn't have direct physical meaning, but its **square** tells us something very important.

• The **square of the wave function**, written as **$|\psi|^2$**, gives the **probability density**.

• This means it tells us how **likely** it is to find the particle at a certain position in space.

• Example: If $|\psi(x)|^2$ is large at some point x, the particle is **more likely** to be found there.

• If $|\psi(x)|^2$ is small or zero, the particle is **less likely** or **not likely at all** to be there.

• The total area under the curve of $|\psi|^2$ (over all space) must equal **1**, which means the particle exists **somewhere** with 100% certainty.

◇ **In short**:

- ψ → Mathematical description of a particle's behavior

- $|\psi|^2$ → Probability of finding the particle at a given place

HEISENBERG UNCERTANITY PRINCIPLE

What is the Heisenberg Uncertainty Principle?

The Heisenberg Uncertainty Principle says that it is **impossible to know exactly both the position and momentum of a particle at the same time**.

In simple words:

If you try to **measure a particle's position very accurately**, you will **know less about its momentum** (speed and direction), and if you measure its **momentum precisely**, you will **know less about its exact position**.

Mathematical Form:

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

Where:

• **Δx** = uncertainty in position

• **Δp** = uncertainty in momentum

• **ħ** = reduced Planck's constant (h/2π)

Key Points:

• It is **not a problem of measurement tools** – it is a fundamental property of quantum nature.

• It shows that **particles behave like waves** — and waves are spread out, not at one point.

• The principle is especially important for **tiny particles** like electrons, not for big objects.

Real-Life Examples:

• **Electron Microscopes**: Use this principle to understand limits of resolution.

• **Quantum Tunneling**: Particle can cross a barrier it shouldn't – because of uncertainty in energy.

• **Stability of Atoms**: Electrons don't fall into the nucleus due to uncertainty in position and momentum.

SCHRODINGER TIME-DEPENDEND AND TIME-INDEPENDENT WAVE EQUATION

What is Schrödinger's Equation?

Schrödinger's equation is a key equation in quantum mechanics. It explains how the **wave function (ψ)** of a particle changes with **time and position**.

• It is like **Newton's laws** in classical physics — but for **quantum particles**.

• It helps us calculate the **behavior and energy** of particles like electrons in atoms.

There are two main forms:

- **Time-Dependent Schrödinger Equation (TDSE)**
- **Time-Independent Schrödinger Equation (TISE)**

8. TIME-DEPENDENT SCHRÖDINGER EQUATION (TDSE)

Equation:

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

Where:

• **ψ(x, t)** = wave function (depends on position and time)

• **i** = imaginary unit (√–1)

• **ħ** = reduced Planck's constant (h/2π)

• **m** = mass of the particle

• **V(x)** = potential energy

Meaning:

• This equation describes how the wave function **evolves over time**.

• It gives a **complete description** of a quantum system.

Derivation (Basic Idea):

• Start with **energy conservation**:

E = K.E. + P.E. = (p^2/2m) + V

• In quantum mechanics, replace:

E → iħ(∂/∂t)

p → –iħ(∂/∂x)

• Plug these into the classical energy equation to get Schrödinger's equation.

$$\hat{E}\psi = \left(\frac{\hat{p}^2}{2m} + V(x)\right)\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)\psi$$

9. TIME-INDEPENDENT SCHRÖDINGER EQUATION (TISE)

Equation:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

Where:

• **ψ(x)** = wave function depending only on position

• **E** = total energy (a constant)

• **V(x)** = potential energy function

• Used when the system is **not changing with time** (steady-state).

Derivation (Basic Idea):

• Assume solution of TDSE is:

ψ(x, t) = ψ(x) × e^(–iEt/ħ)

• Substitute into TDSE.

• After separating variables, we get the **TISE**.

10. APPLICATIONS OF SCHRÖDINGER'S EQUATION

Where is it used?

Schrödinger's equation helps explain many quantum systems:

• **Particle in a Box** – explains quantized energy levels

• **Hydrogen Atom** – predicts electron orbitals and energy levels

• **Quantum Tunneling** – used in semiconductors and scanning tunneling microscopes

• **Atoms and Molecules** – used to understand bonding and electronic structure

• **Lasers and LEDs** – designed based on quantum energy transitions

• **Nuclear and Particle Physics** – understanding behavior of subatomic particles

PARTICLEIN POTENTIA 1 1-DBOX

What is a Particle in a 1D Box?

It is a basic quantum model where a particle (like an electron) is **trapped inside a box** (a region with fixed boundaries) and

cannot escape.

• The box has **walls of infinite potential**, so the particle is **only free inside the box**.

• Outside the box, the particle's wave function is **zero**.

• It helps us understand **quantum energy levels** and **wave behavior**.

Assumptions:

• The box has **length L** (from x = 0 to x = L).

• Potential **V = 0** inside the box (0 < x < L), and

V = ∞ outside the box.

• The particle is **free to move** inside the box but **cannot exist** outside.

Schrödinger's Equation (inside the box):

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

This is the **Time-Independent Schrödinger Equation** for this system.

Boundary Conditions:

$$\psi(0) = 0, \quad \psi(L) = 0$$

(Since the particle cannot exist at the walls)

Solution (Wave Function):

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$

Where **n = 1, 2, 3,...** (quantum number)

Energy Levels:

$$E_n = \frac{n^2h^2}{8mL^2}$$

Where:

• **n** = 1, 2, 3,...

Assume a **separable solution:**

$$\psi(x,t) = \psi(x)\cdot T(t)$$

1. Substitute into TDSE:

$$i\hbar\frac{d}{dt}[\psi(x)T(t)] = \left(-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x)\right)T(t)$$

$$i\hbar \frac{d}{dt}[\psi(x)T(t)] = \left(-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x)\right)T(t)$$

2. Divide both sides by $\psi(x)T(t)$:

$$\frac{i\hbar}{T(t)}\frac{dT(t)}{dt} = \frac{1}{\psi(x)}\left[-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x)\right]$$

• As **n increases**, the energy and number of wave peaks increase.

Applications:

• Explains **quantum confinement** in nanotechnology.

• Helps understand behavior of **electrons in atoms and quantum dots**.

• Basis for learning more complex quantum systems.

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